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Methodical Solutions of Bacteria and Tumour Growth Models via Tarig Transform

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ABSTRACT: The integral transforms are suitable for mathematical and physical application. Integral transforms are extremely effective mathematical methods for addressing a wide range of complex issues in science and engineering, including issues with population growth, heat conduction, motion of particles under gravity, vibration of beams, and radioactive decay. The solution of various engineering and scientific problems can be easily determined by representing these problems in integral equations. There are numerous analytical and numerical methods available for solving various types of integral equations. In this paper, we find exact solutions of bacteria growth and Tumoursize growth Models through an Integral transformation, namely the Tarig Transform. For demonstrating the effectiveness of Tarig transform, we consider some numerical examples. The results of these numerical examples shows that Tarig transform provides the analytical solution of bacteria growth and tumour size growth models without doing complicated computational work.It has been revealed that the Tarig transform is a practical, dependable, and simple technique for obtaining solutions to the growth problems.

Keywords: Tarig Transform, Inverse Tarig Transform, Differential Equations, Bacteria Growth and Tumour Growth Models.

I. INTRODUCTION:

Now a day, Integral transforms are the best appropriate and easy mathematical process for finding advance problems solutionarose in several fields like technology, science, social sciences, commerce, economics and engineering. Integral transforms provide exact solution of problem without lengthy calculations that is the vital feature of integral transforms.Due to this vital feature of the integral transforms various investigators are involved to this field and acquaint with many integral transforms. Differential equations are involved to examine the real-life problems; including Biological growth, Tumour growth, Heat transfer, Carbon dating, Compound interest, Chemical reaction, Mixture, Compartment, Electric Circuit, trajectory and vibrations problems [1].Further most problems in the seareas are modeled via ordinary linear differential equations and made more reasonable. There occur numerous mathematical and analytical methods in the literature for solving the different forms of differential equations [2-8].Afterward, integral transforms methodologies provide accurate solutions of the problems thus various researchers are developing new integral transforms [9-17].

The study of growth problem is one of the challenging problems in many areas. Growth problems can be usually used in the field of sciences, social science and among other subjects. Various masses in the real-worlds growth at a quantity proportional to their size. Various integral transforms have been solved the population growth problems. As various investigators involved to presenting the new integral transforms at the same time and as well applying the transforms to various fields, various equations in different domain. Cooling law of Newton's problem was solved by Sanap and Patil [18], with the help of Kushare transform.Tarig M.Elzaki and Salih M. Elzaki [20-22], introduced new transform known as Tarig transform and studied for finding the solution of the application of differential equations. Gnanavel et al.[23], found the solutionof the applications of linear Volterra integral equation of first kind by using Tarig transform. Aggarwal and other scholars [24-33], studied the growth and decay models using various integral transformations such as Laplace transform, Mohand transform, Kamal transform, Aboodh transform, Mahgoub transform, Sadik transform, Elzaki transform, Shehu transform, Sumudu transform and Sawi transform. Aggarwal and other scholars [34-39], comparatively studied various integral transformations and Mohand transform and solved the system of ordinary differential equations using them. Patil [40], have been used (Laplace and Shehu) transforms to gain the solution of chemical science problems. Deshmukh et al. [41], utilized Emad Sara transform to solve the problems related to population growth and decay. Dinesh Thakur and P.C. Thakur [42], Employing Upadhyaya Transform for finding the solution of linear second kind Volterra Integral equation. Thakur et al. [43], studied the linear Volterra integral equation (V.I.E) by employing Iman transform. Patil et al. [44], used the Applications of Karry-Kalim-Adnan Transformation (KKAT) in growth and decay problems. Recently, Aggarwal [45], obtained the solution of the Bacteria Growth Model via Rishi Transform.

Growth Model: Mathematically, the equation of growth is a first order linear ordinary differential equation. Growth can be expressed as the first order derivative of amount of physical material $M(t)$ is directly proportional to quantity of physical material $M(t)$ at time t hour. The living growth such as growth of plant, growth of bacteria, growth of a species, growth of cell, growth of an organ etc. are governed by linear ordinary differential equation of first order as below:

$$\frac{dM(t)}{dt} \alpha M(t)$$

Therefore,

$$\frac{dM(t)}{dt} = \xi M(t),$$

through the initial Condition $M(0) = M_0$ at time $t = 0$

(1)

where, $M(t)$ and M_0 are the quantity of physical material at the time t and $t = 0$, that is the exponential growth at rate proportional to its quantity material. ξ be the proportionality rate andthe equation (1) is called the act of Natural Growth Model.

The main purpose of this paper is to determine the solution of the bacteria growth and tumour growth models using newly established Tarig transform Technique and its efficiency to solve bacteria growth and tumour growth problems effectively.

II. TARIG TRANSFORM,ITS PROPERTIES AND TABULATED VALUES[20 - 23]

A new integral transform introduced and studied by Elzaki et al.called Tarig transform. We consider function $f(t)$, in the form of set A, which is expressed in exponential form as below: [7],

$$A = \left\{ f(t) : \exists M, \lambda_1, \lambda_2 > 0, |f(t)| < M e^{\frac{|t|}{\lambda_j}}, if\ t \in (-1)^j X[0,\infty) \right\}$$

(2)

where, M be a finite number that is constant and λ_1 *and* λ_2 may be finite and infinite.

2.1. Tarig Transform Definition [20-22]

2.1A. Definition of Tarig Transform: Tarig transform is a form of integral transform that presented and deliberated by Elzaki et al., Tarig transform symbolized by $T[.]$, defined below as:

$$T[f(t)] = \frac{1}{v}\int_0^\infty e^{-\left(\frac{t}{v^2}\right)} f(t)dt = B(v) \qquad t \geq 0, \quad v \neq 0$$

(3)

2.1B. Definition of Inverse Tarig Transform: Inverse of Tarig transform from equation (3), defined as below:

$$f(t) = T^{-1}\left\{\frac{1}{v}\int_0^\infty e^{-\left(\frac{t}{v^2}\right)} f(t)dt\right\} = T^{-1}[B(v)], \quad t \geq 0, \quad v \neq 0$$

(4)

with the help of Tarig transform, we can easily solve the mathematical models in health sciences, environmental sciences and biochemistry, containing ordinary linear differential equation of first order. The aim of this study is to shows the applicability of this interesting transform and operator $B(v)$ defined by the above integral equations.

International Journal of Advances in Engineering and Management (IJAEM)
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1.2. Derivative of Tarig Transform:

2.2A. Ordinary Derivative of Tarig Transform: [21]

$$\left.\begin{aligned}
&\text{First derivative:} \quad T\left[f'(t)\right] = \frac{T\ f(t)}{v^2} - \frac{1}{v}\ f(0) = T\left[\frac{df(t)}{dt}\right] \\
&\text{Second derivative:}\ T\left[f''(t)\right] = \frac{T\left[f(t)\right]}{v^4} - \frac{1 f(0)}{v^3} - \frac{1}{v}\ f'(0) = T\left[\frac{d^2 f(t)}{d t^2}\right] \\
&\text{nth derivative:}\ T\left[f^{n}(t)\right] = \frac{T\left[f(t)\right]}{v^{2n}} - \sum_{i=1}^{n} v^{2(i-n)-1}\ f^{(i-1)}(0) = T\left[\frac{d^n f(t)}{d t^n}\right]
\end{aligned}\right\} \qquad (5)$$

2.2.B. Tabulated Values: [23]

Chart for Tarig transform and Inverse of Tarig transform that often used for frequently encountered mathematical functions are given in Table -2.2.B_1 and Table -2.2.B_2.

Table 2.2.B_1: Standard Results

Function $f(t)$	1	t	t^2	$\frac{t^n}{n!}$	e^{at}	e^{-at}	$\sin at$	$\sinh at$	$\cos at$	$\cosh at$
$B(v)=T[f(t)]$	v	v^3	$2v^5$	$\frac{1}{v^{-(2n+1)}}$	$\frac{v}{1-av^2}$	$\frac{v}{1+av^2}$	$\frac{av^3}{1+(av^2)^2}$	$\frac{av^3}{1-(av^2)^2}$	$\frac{v}{1+(av^2)^2}$	$\frac{v}{1-(av^2)^2}$

Inverse Tarig Trans form of function $f(t)$ defined as $f(t) = T^{-1}\{B(v)\}$ and Some Standard results of Inverse Tarig transform of the functions are listed in Table 2.2.B_2

Table 2.2.B_2: Standard Results

$B(v)=T[f(t)]$	v	v^3	$2v^5$	$\frac{1}{v^{-(2n+1)}}$	$\frac{v}{1-av^2}$	$\frac{v}{1+av^2}$	$\frac{av^3}{1+(av^2)^2}$	$\frac{av^3}{1-(av^2)^2}$	$\frac{v}{1+(av^2)^2}$	$\frac{v}{1-(av^2)^2}$
$f(t) = T^{-1}\{B(v)\}$	1	t	t^2	$\frac{t^n}{n!}$	e^{at}	e^{-at}	$\sin at$	$\sinh at$	$\cos at$	$\cosh at$

III. BACTERIA GROWTH AND TUMOURSIZE GROWTH MODELS

Consider the Malthus model [25-29] for the significance of the growth of the bacteria and tumour size in a certain culture consulting to Malthus model, at which bacteria grow and tumour size grow rate in a certain culture is proportional to the quantity of bacteria present and tumoursize present at the time t. Generally bacteria growth and tumour size growth problems expressed as the rate proportional to the amount of bacteria and tumour size $M(t)$, subsequently time t hours in the first order form of differential equation,

Mathematically, bacteria growth and tumour size growth model defined as below from the equation (1)

$\frac{d M(t)}{dt} = \xi M(t)$; through the condition $M(0) = M_0$ at time $t = 0$.

(6)

The equation of bacteria growth and tumour size growth (6) is a first order linear ordinary differential equation. Where, $M(t)$ and M_0 are the quantity of bacteria and tumour size at the time t and time $t=0$, which are in the nature of exponential growth at rate proportional to its quantity of bacteria and tumour size. ξ be the proportionality rate and the equation (6) is called the act of natural bacteria growth and tumour size growth.

IV. TECHNIQUE:

Tarig transform technique for finding the solution of Bacteria growth and Tumour Size growth Models.

Relating the Tarig transform to the exponential growth model given in equation (6) both the sides, we get

$$T\left[\frac{dM(t)}{dt}\right]=T[\xi M(t)] \qquad (7)$$

Substituting the Tarig transform of the first derivative value from equation (5) in equation (7), we obtain

$$\frac{B(v)}{v^2}-\frac{M(0)}{v}=\xi.B(v) \qquad (8)$$

Using the condition that at a time $t=0$, the quantity of bacteria and tumour be $M(0)=M_0$, in equation (8) and after simplification, we obtain

$$B(v)=M_0\left(\frac{v}{1-\xi v^2}\right) \qquad (9)$$

Operating inverse Tarig Transform jointly to the equation (9) and using the table-2.2.B_2, we obtain

$$T^{-1}[B(v)]=M_0T^{-1}\left(\frac{v}{1-\xi v^2}\right)$$

$$M(t)=M_0e^{\xi t} \qquad (10)$$

which is the required number of bacteria and tumour size in a certain culture at time t. Tarig transform technique be one of the type of integral transform that provides abundant suitability in solving first order differential equations and obtain solution accurately that is coinciding with the existing result obtained by using the other integral transform.

V. NUMERICAL APPLICATIONS:

In this fragment, Tarig transform technique have been applied to find the solution of the general form of bacteria growth and tumour size growth model. Some applications have been used to establish the effectiveness of Tarig transform technique.

5.A. BACTERIA GROWTH:

Application-5.A_1 :

Bacteria in a certain culture increases at a rate proportional to the number present. If the number of bacteria increases from 1000 to 2000 in one hour, estimate the number of bacteria present in a certain culture at the end of 1.5 hours.

Above stated bacteria growth application, mathematically, be expressed at rate proportional to the number of bacteria present in a certain culture as below [45];

$\frac{dM(t)}{dt}=\xi M(t)$; through the initial condition

$$M(0)=M_0 \text{ at } t=0 \qquad (11)$$

Here, constant of proportionality be denoted by ξ and the number of bacteria at time t and $t=0$ be denoted by M and M_0.

Relating Tarig Transform to the equation (11) both sides, we obtain

$$T\left[\frac{dM(t)}{dt}\right]=T[\xi M(t)] \qquad (12)$$

Substituting the Tarig Transform values of the first derivative from equation (5) in equation (12), we obtain

$$\frac{B(v)}{v^2}-\frac{M(0)}{v}=\xi.B(v) \qquad (13)$$

Using the condition that at a time $t=0$, the quantity of bacteria be $M(0)=M_0=1000$, in equation (13) and after simplification, we obtain

$$\frac{B(v)}{v^2}-\frac{M_0}{v}=\xi.B(v)$$

$$\frac{B(v)}{v^2}-\frac{1000}{v}=\xi.B(v) \qquad (14)$$

After re-arranging, we obtain

$$B(v)=1000\left(\frac{v}{1-\xi v^2}\right) \qquad (15)$$

Operating Inverse Tarig transform jointly to the equation (15) and using the table-2.2.B_2,we obtain

$$T^{-1}[B(v)]=1000T^{-1}\left(\frac{v}{1-\xi v^2}\right)$$

$$M(t)=1000\ e^{\xi t} \qquad (16)$$

Also, attime $t=0$ and $t=1$, the number of

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bacteria are $M(0) = M_0$ and $M(2) = 2000$; Substituting these values in equation (16), we obtain

$$2000 = 1000\ e^{\xi}$$

$$\Rightarrow \quad e^{\xi} = 2$$

$$\Rightarrow \quad \xi = \ln(2) = 0.693$$

Hence, $\xi = 0.693$ (17)

Again, at time $t = 1.5$, the number of bacteria $M(1.5)$ present in a certain culture be obtain by substituting the values of t, ξ and M in the equation $M(t) = M_0 e^{\xi t}$.

Therefore, $M(1.5) = 1000\, e^{1.5(0.693)}$;

$$\Rightarrow \quad M(1.5) \approx 2825.70;$$

Hence, $M(1.5) = 2825.70$. (18)

which is the required number of bacteria present in a certain culture at the time t.

This obtain solution by using Tarig transform technique is nearly coinciding with the existing result obtained by using the Rishi Transform [45].

Application-5.A$_2$:

Bacteria in a certain culture rises at a rate proportional to the quantity of bacteria presently living in a certain culture. If after two years, bacteria in a certain culture have doubled, and after three years' bacteria in a certain culture is 20,000, estimate the number of bacteria initially in a certain culture.

Above stated bacteria growth application, mathematically be expressed as the rate proportional to the number of bacteria as below [45];

$\frac{d\,M(t)}{dt} = \xi\, M(t)$; through the condition

$$M(0) = M_0 \text{ at } t = 0 \qquad (19)$$

Here, constant of proportionality be denoted by ξ and the number of bacteria at time t and $t = 0$ be denoted by M and M_0.

Relating Tarig transform to the equation (19) both sides, we obtain:

$$T\left[\frac{d\,M(t)}{dt}\right] = T[\xi\, M(t)] \qquad (20)$$

Substituting the Tarig transform values of the first derivative from equation (5) in equation (20), we obtain

$$\frac{B(v)}{v^2} - \frac{M(0)}{v} = \xi . B(v) \qquad (21)$$

Using the condition that at time $t = 0$, the quantity of bacteria be $M(0) = M_0$ in equation (21) and after simplification, we obtain

$$\frac{B(v)}{v^2} - \frac{M_0}{v} = \xi . B(v) \qquad (22)$$

After re-arranging, we obtain

$$B(v) = 1000\left(\frac{v}{1 - \xi v^2}\right) \qquad (23)$$

Operating Inverse Tarig transform jointly to the equation (23) from the table 2.2.B$_2$, weobtain

$$T^{-1}[B(v)] = 1000 T^{-1}\left(\frac{v}{1 - \xi v^2}\right)$$

$$M(t) = M_0\, e^{\xi t} \qquad (24)$$

Also, at the time $t = 2$, the number of bacteria be $M(2) = 2M_0$;

Substituting these values in equation (24), we obtain

$$2M_0 = M_0\, e^{2\xi}$$

$$\Rightarrow \quad e^{2\xi} = 2$$

$$\Rightarrow \quad \xi = 0.3466$$

Hence, $\xi = 0.3466$ (25)

Again, atthe time $t = 3$, the number of bacteria be $M(3) = 20{,}000$; substituting the values of t, ξ and M in equation $M(t) = M_0 e^{\xi t}$, we obtain

$$20{,}000 = M_0 e^{3(0.346)};$$

$$20{,}000 = M_0 (2.82647);$$

Hence, $M_0 = 7075$. (26)

which is the required number of bacteria initially in a certain culture time t. this obtain solution by using Tarig transform technique is nearly coinciding with the existing result obtained by using the Rishi Transform [45].

5.B. TUMOUR GROWTH MODEL

Application-5. B$_1$:

One model used in medicine is that the rate of growth of tumour is proportional to the size of the tumour. Write a differential equation

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satisfied by M, the size of tumour in mm as a function of time t, the tumour is 5mm across at time $t=0$. Find the solution in addition if the tumour is 8mm across at time $t=3$, find particular solution.

Above stated growth problem mathematically, be expressed at rate proportional to the size of tumour as below [44];

$\frac{dM(t)}{dt} = \xi\, M(t)$; through the initial condition

$$M(0) = M_0 \text{ at } t = 0 \qquad (27)$$

Here, constant of proportionality be denoted by ξ and the size of tumour at time t and $t=0$ be denoted by M and M_0.

Relating Tarig transform to the equation (27) both sides, we obtain

$$T\left[\frac{dM(t)}{dt}\right] = T[\xi M(t)] \qquad (28)$$

Substituting the Tarig transform values of the first derivative from equation (5) in equation (28), we obtain

$$\frac{B(v)}{v^2} - \frac{M(0)}{v} = \xi . B(v) \qquad (29)$$

Using the condition that at a time $t=0$, the size of tumour be $M(0) = M_0$ in equation (29) and over simplification, we obtain

$$\frac{B(v)}{v^2} - \frac{M_0}{v} = \xi . B(v) \qquad (30)$$

After re-arranging, we obtain

$$B(v) = M_0\left(\frac{v}{1-\xi v^2}\right) \qquad (31)$$

Operating Inverse Tarig transform jointly to the equation (31) from the table 2.2.B_2, we obtain

$$T[B(v)] = M_0 T\left(\frac{v}{1-\xi v^2}\right)$$

$$M(t) = M_0 e^{\xi t} \qquad (32)$$

Also, at $t=0$, the size of tumour be $M_0 = 5mm$;

Substituting these values in equation (32), we obtain

$$M(t) = 5e^{\xi t} \qquad (33)$$

Again, at a time $t=3$, the size of tumour be $M(3) = 8mm$; substituting the values of t and M in equation (33), we obtain

$$8 = 5e^{3\xi};$$

$$\Rightarrow \quad e^{3\xi} = \frac{8}{5}$$

$$\Rightarrow \quad \xi = \frac{1}{3}\ln(1.6) \cong 0.1567$$

Putting the value of $t=3$ and $\xi = 0.1567$ in equation $M(t) = 5e^{\xi t}$, we get

$$M(t) = 5e^{3(0.1567)} = 5e^{0.4701}$$

$$M(t) = 5(1.60054201) = 6.3893$$

$$M(t) = 6.3893$$

Hence, $M(t) = 6.3893$ (34)

which is the required particular solution.

This obtain solution by using Tarig transform technique is nearly coinciding with the existing result obtained by using the (KKAT) Transform [44].

VI. DISCUSSION AND CONCLUSION:

- In this work, we have successfully applied Tarig transform technique for finding the solution of bacteria growth and tumour growth models.
- Applicability and competency of Tarig transform is demonstrated by giving some mathematical application of bacteria growth and tumour growth models.
- We observed that the result depict that the Tarig transform is a very efficient integral transform for solving the application of bacteria growth and tumour size growth models.
- Tarig transform provides the analytical solution of the application of bacteria growth and tumour size growth models without doing complicated calculation work as compared to other integral transform.
- In future, the suggested scheme can be applied for determining the solutions of radioactive substance decay model, model of chemical kinetic, traffic model, electric circuit model, compartment models, diabetes detection model, compound interest and heat conduction problems related to various different fields.

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